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η_i-mode in toroidal plasmas with anisotropic ion temperature and its gradient

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Abstract

The gyrokinetic integral eigenmode equation is applied to study ion-temperature-gradient (ITG) mode in toroidal plasmas with magnetic shear, where the ion temperature and its gradient are anisotropic. The numerical studies demonstrate that instability of ITG mode is reduced by an ion temperature anisotropy of higher perpendicular temperature $T_{i\perp}$ or high enough parallel temperature $T_{i\|}$, in which Landau damping plays an important role. The temperature gradient in the perpendicular (parallel) direction is stronger to drive ITG mode for large $T_{i\perp}$ ($T_{i\|}$). These effects are directly related to the temperature gradient threshold for excitation of ITG mode. In addition, the synergy effect of magnetic shear and anisotropy of ion temperature and its gradient is studied in detail, where the combination of magnetic shear and large parallel temperature has the most obvious inhibitory effect on ITG mode.

Keywords: gyro-kinetic equation, ITG mode, ion temperature and temperature gradient anisotropy

(Some figures may appear in colour only in the online journal)

1. Introduction

The micro-drift instability has always been the focus of fusion research since it induces anomalous particle, energy and momentum transports in tokamaks [1, 2]. The plasma ion temperature and its gradient are considered isotropic in most theoretical studies. However, with the wide use of auxiliary heating, such as ion cyclotron resonance frequency heating and neutral beam injection, the plasmas anisotropy is more and more obvious in the present fusion experimental machine, as well as the future demo. Therefore, it is necessary to consider the effects of such anisotropy on the drift wave instabilities responsible for anomalous transport in more detail.

The first investigation of ion temperature anisotropy on ion temperature gradient mode can be traced back to Migliuolo's work [3] in 1988, he pointed that the anisotropy gave an overall stabilizing effect in a shearless slab geometry when perpendicular temperature is greater than parallel temperature. Then, Mathey and Sen [4] used the local kinetic theory to study the effect of anisotropy of ion temperature and its gradient on ITG mode in a shearless slab configuration, and applied the theory to the experiments on the Columbia linear machine. They showed that excitation of ITG mode needs a finite parallel temperature gradient, and a gradient in the perpendicular temperature can either enhance or diminish the instability. Kim *et al* [5] developed the work for a shearless toroidal geometry with a local kinetic theory. Comparing with the results in a slab geometry, the opposite conclusion in a toroidal system was obtained, where the gradient in the perpendicular temperature cannot stabilize ITG mode, but induces destabilization of the ITG mode just like the parallel temperature gradient does. Later, Song and Sen [6] considered the collision effects, using local kinetic theory, which means that the magnetic shear effect is hardly to be taken into account under the nonlocal consideration, although a small shear limit was considered in the work. Furthermore, Dong *et al* [7] studied the ion temperature anisotropy effect in a sheared slab geometry using fluid and kinetic theories, where the anisotropy in ion

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temperature gradient enhanced (reduced) the stabilization effect from a magnetic shear for $\eta_\perp > \eta_\parallel (\eta_\perp < \eta_\parallel)$. An anisotropy of $T_\perp > T_\parallel$ in ion temperature is found to give an overall stabilization (destabilization) for low (high) magnetic shear $s \sim 0.1$ ($s \sim 0.4$).

In this paper, we extent the work to toroidal plasmas with magnetic shear. An integral eigenvalue equation, keeping anisotropy of ion temperature and its gradient, magnetic shear effect, full finite Larmor radius effects, magnetic gradient and curvature drifts, and resonant wave-particle interaction, is derived from the linear gyrokinetic equation in electrostatic limit for the ITG mode. The equation is solved numerically with the HD7 code [8] and the results are presented in detail. The synergy of the magnetic shear and the anisotropy of temperature and its gradient is the main focus.

The remainder of this paper is organized as follows. The integral eigenvalue equation and physical model are presented in section 2. The numerical results are shown and analyzed in section 3. Finally, the brief conclusions are drawn in section 4.

2. Integral eigenvalue equation

In a tokamak configuration, the typical magnetic field is given as

$$\mathbf{B} = \frac{rB_0}{Rq}\mathbf{e}_\theta + B_0\mathbf{e}_\xi, \tag{1}$$

where r, θ and ξ are the radial, poloidal and toroidal directions, respectively. q is the safety factor, R is the major radius of the torus, B_0 is the toroidal magnetic field.

The basic equation for the study of low-frequency electrostatic perturbations in inhomogeneous plasmas is the quasineutrality condition, $\tilde{n}_e = \tilde{n}_i$. Here, adiabatic electron response to the electrostatic perturbation is assumed, and the perturbed ion density in an axisymmetric toroidal geometry is obtained straightly from the gyrokinetic equation [8]. The ballooning mode representation [9] is used to derive the eigenvalue equation. The equilibrium Maxwell distribution is written as

$$F_{\mathrm{Mi}} = \left(\frac{m_i}{2\pi}\right)^{\frac{3}{2}} \frac{n_{0i}}{T_{i\perp}\sqrt{T_{i\parallel}}} \exp\left(-\frac{m_i}{2T_{i\perp}}\mathrm{v}_\perp^2 - \frac{m_i}{2T_{i\parallel}}\mathrm{v}_\parallel^2\right), \tag{2}$$

where $T_{i\perp}$ and $T_{i\parallel}$ are the ion temperatures in the directions perpendicular and parallel to the magnetic field, respectively. We define an ion temperature anisotropy parameter

$$\Lambda_i = \frac{T_{i\perp}}{T_{i\parallel}} - 1.$$

When $\Lambda_i > 0$, it presents that the perpendicular ion temperature $T_{i\perp}$ is higher than the parallel temperature $T_{i\parallel}$. $\Lambda_i < 0$ indicates a higher parallel temperature, and $\Lambda_i = 0$ means isotropic plasma temperature. For simplicity, we ignore trapped particle contribution and $\mathrm{v}_\parallel$ modulation along the unperturbed particle orbit for passing particles. From the quasineutrality condition, the Fredholm homogeneous integral equation of the second kind [10, 11] can be obtained. The method induced by Dong *et al* [8] is also employed in the derivation, where the integration over the perpendicular velocity is performed analytically, while the integration over the parallel velocity is converted to an integration over time. Finally, we obtain the integral eigenvalue equation for ITG mode with ion temperature and its gradient anisotropy in tokamak plasmas as

$$(1 + \tau_e + \Lambda_i\tau_e\Gamma_0)\hat{\phi}(k) = \int_{-\infty}^{\infty} \mathrm{d}k' K(k,k')\hat{\phi}(k'), \tag{3}$$

where

$$\begin{aligned} K(k,k') = &-\frac{\mathrm{i}}{2\sqrt{\pi}}\int_{-\infty}^{0} \omega_{*e}\,\mathrm{d}t\,\frac{\mathrm{e}^{-\mathrm{i}\omega t}}{\lambda\sqrt{a\zeta}}\,\mathrm{e}^{-(k-k')^2/4\zeta} \\ &\times\left\{\frac{\omega\tau_e}{\omega_{*e}} + 1 - \eta_\perp - \frac{1}{2}\eta_\parallel + \frac{\eta_\perp}{\lambda} + \eta_\parallel\frac{(k-k')^2}{4a\zeta}\right. \\ &\left. - \eta_\perp\frac{k_\perp^2 + k_\perp'^2}{4\lambda^2\tau_e} + \frac{\eta_\perp k_\perp k_\perp'}{2\lambda^2\tau_e}\frac{I_1}{I_0}\right\}\Gamma_0(k_\perp, k_\perp'), \end{aligned}$$

and

$$\begin{aligned} a = &\,1 + \mathrm{i}\frac{2\varepsilon_n\omega_{*e}t}{(1+\Lambda_i)\tau_e} \\ &\times\left[\frac{(1+s)(\sin\theta - \sin\theta') - s(\theta\cos\theta - \theta'\cos\theta')}{\theta - \theta'}\right] \end{aligned}$$

$$\lambda = \frac{1 - \Lambda_i + a(1+\Lambda_i)}{2}, \qquad \zeta = \frac{\omega_{*e}^2 t^2}{(1+\Lambda_i)\tau_e a}\left(\frac{s}{q}\varepsilon_n\right)^2,$$

$$\theta = \frac{k}{sk_\theta}, \quad \theta = \frac{k'}{sk_\theta}, \quad k_\perp^2 = k_\theta^2 + k^2, \quad k_\perp'^2 = k_\theta^2 + k'^2,$$

$$\Gamma_0 = I_0\left(\frac{k_\perp k_\perp'}{2\lambda\tau_e}\right)\mathrm{e}^{-(k_\perp^2 + k_\perp'^2)/4\lambda\tau_e}, \quad L_{T\parallel} = -\left(\frac{\mathrm{d}\ln T_{i\parallel}}{\mathrm{d}r}\right)^{-1}$$

$$L_n = -\left(\frac{\mathrm{d}\ln n}{\mathrm{d}r}\right)^{-1}, \quad L_{T\perp} = -\left(\frac{\mathrm{d}\ln T_{i\perp}}{\mathrm{d}r}\right)^{-1}, \quad \tau_e = \frac{T_e}{T_{i\perp}}$$

$$\eta_\perp = \frac{L_n}{L_{T\perp}}, \quad \eta_\parallel = \frac{L_n}{L_{T\parallel}}, \quad \varepsilon_n = \frac{L_n}{R}, \quad \omega_{*e} = \frac{ck_\theta T_e}{\mathrm{e}BL_n}.$$

Here, ω_{*e} is the electron diamagnetic drift frequency, k is the Fourier transform of radial variable x, the magnetic shear is $s = (r/q)(\mathrm{d}q/\mathrm{d}r)$, L_n and L_T are the density and temperature gradient scale lengths, respectively. $\eta_\perp$ and $\eta_\parallel$ present the ion temperature gradient parameters in the perpendicular and parallel magnetic field directions, respectively. $I_j (j = 0, 1)$ is the modified Bessel function of order j. Also the wave number k, k' and k_θ are normalized to ρ_s^{-1} with $\rho_s = \sqrt{2T_e/m_i}/\Omega_i$ and $\Omega_i = \mathrm{e}B/m_ic$, $\mathrm{v}_\perp$ and $\mathrm{v}_\parallel$ are normalized to the ion thermal velocity in the perpendicular $\mathrm{v}_{T\perp} = \sqrt{2T_{i\perp}/m_i}$ and parallel $\mathrm{v}_{T\parallel} = \sqrt{2T_{i\parallel}/m_i}$ directions, respectively. The electric potential perturbation ϕ is normalized to $\mathrm{e}\phi/T_{i\parallel}$.

The integral eigenmode equation includes full kinetic mechanisms of the ions, such as magnetic shear, magnetic gradient and curvature drifts, and transit motion along magnetic field lines.

3. Numerical results

Computer code HD7 using Raleigh–Ritz method [12] to solve the Fredholm homogeneous integral equation of the second kind, has been widely used in the study of micro-drift instability in slab [13, 14], tokamak [8, 15] and reversed-field pinch [16, 17] plasmas. In addition, it has been well benchmarked with other simulation codes [18], such as GTC, GT3D and XGC1. On the other hand, most of the previous works focus on the case that the perpendicular temperature is higher than the parallel temperature ($\Lambda_\mathrm{i} > 0$) and rarely involves the case of $\Lambda_\mathrm{i} < 0$. Taking into account the anisotropy of ion temperature and its gradient, the code HD7 is upgraded in accordance with equation (3) in this work. Thus, the influence of the anisotropy of ion temperature and its gradient on ITG mode is able to be investigated numerically in detail.

3.1. Synergy of magnetic shear and ion temperature anisotropy

Firstly, we study the effect of ion temperature anisotropy and neglect the ion temperature gradient anisotropy for simplicity. The normalized growth rate and real frequency of ITG mode versus temperature anisotropy parameter Λ_i are plotted in figure 1 for different magnetic shear. The parameters are $\eta_\perp = \eta_\parallel = 3.0, \varepsilon_n = 0.2, k_\theta = 0.45, q = 1.5$ and $\tau_\mathrm{e} = 1.0$. From the figure, one can see that the ITG mode becomes stable with the increasing of Λ_i values when the perpendicular temperature is greater than the parallel temperature ($T_{\mathrm{i}\perp} > T_{\mathrm{i}\parallel}$), which means $\Lambda_\mathrm{i} > 0$. This result may be due to the bigger Λ_i value reducing the real frequency of ITG mode, which leads to the stronger Landau damping. Therefore, the large Λ_i value can reduce the instability of ITG mode. This conclusion is consistent with the previous work in slab or local toroidal geometry. However, ITG mode also can be stabilized when $T_{\mathrm{i}\perp} < T_{\mathrm{i}\parallel}$, $|\Lambda_\mathrm{i}| \geqslant 0.5$ in our cases. This may also be caused by Landau damping. Although the increase of negative Λ_i increases the drift frequency and thus the real frequency of the mode, reducing Landau damping, it also corresponds to higher parallel temperature and higher parallel thermal velocity of the ions. The increase of parallel velocity also means increase of Landau damping. Therefore, the behavior of the mode is result of competition between the two mechanisms and the latter may produce stronger Landau damping and stabilizing effect on the modes when the negative Λ_i increases to a certain value. In addition, the stable and unstable turning points change with magnetic shear. The bigger magnetic shear value, the smaller the turning point $|\Lambda_\mathrm{i}|$ value is. Moreover, it should be pointed out that the magnetic shear $s = 0.1$ stabilizes ITG mode more effectively and its stabilizing effect even exceeds that of large magnetic shear ($s = 1.5$). This result is in agreement with the result in a slab configuration including magnetic shear effect [7], the lower magnetic shear ($s \sim 0.1$) can give an overall stabilization effect when an anisotropy of $T_{\mathrm{i}\perp} > T_{\mathrm{i}\parallel}$ is considered.

The effect of magnetic shear on temperature anisotropy is further studied in figure 2, where the normalized mode growth rate versus the normalized poloidal wave number k_θ is presented for $s = 0.1$ and $s = 0.4$. The results show that

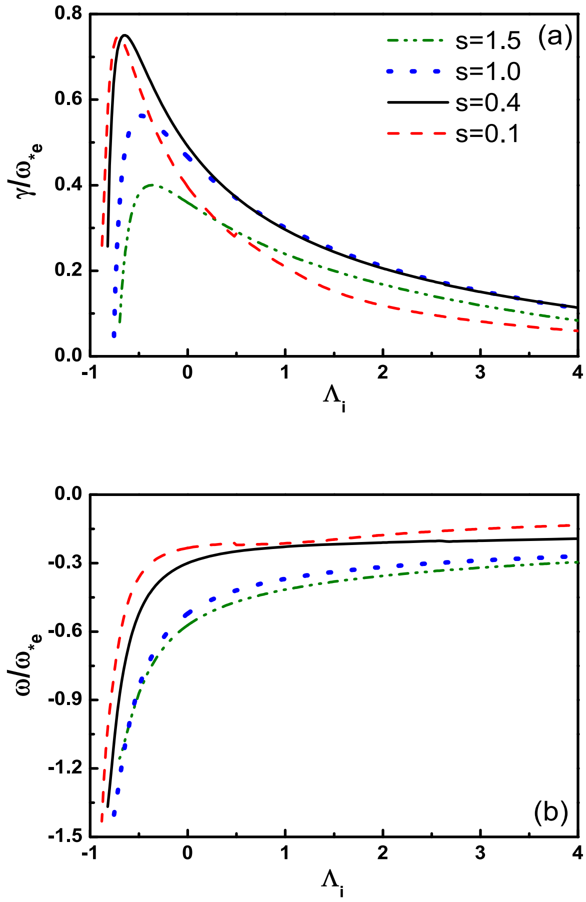

Figure 1. Normalized growth rate $\gamma/\omega_{*\mathrm{e}}$ and real frequency $\omega/\omega_{*\mathrm{e}}$ vs Λ_i for different s. The other parameters are $\eta_\perp = \eta_\parallel = 3.0, \varepsilon_n = 0.2, k_\theta = 0.45, q = 1.5, \tau_\mathrm{e} = 1.0$.

the growth rate is the highest, intermediate and the lowest when $\Lambda_\mathrm{i} = 0, 0.5$, and 1.0, respectively. This means that the anisotropy in the direction of $T_{\mathrm{i}\perp} > T_{\mathrm{i}\parallel}$ gives an overall stabilizing effect whatever the magnetic shear value is. This conclusion is different from those in the slab geometry, where the numerical results in figure 7 of reference [7] show that the anisotropy in the direction of $T_{\mathrm{i}\perp} > T_{\mathrm{i}\parallel}$ has an overall destabilizing effect for $s \geqslant 0.4$. On the other hand, the numerical results show that the stabilization effect is stronger on a short-wavelength modes when $T_{\mathrm{i}\perp} < T_{\mathrm{i}\parallel}$ ($\Lambda_\mathrm{i} = -0.5$).

Shown in figure 3 are the normalized growth rate versus k_θ with different Λ_i. In this figure, a new normalization is employed, where the growth rate is normalized as $\gamma k_\theta/\omega_{*\mathrm{e}} \sim \gamma L_n/c_s$ with $c_s = \sqrt{2T_\mathrm{e}/m_\mathrm{i}}$, in order to compare with the results in a toroidal plasmas without magnetic shear [5]. The same parameters as in figure 2 of reference [5] are used, except for $q = 1.5$ and $s = 0.4, 1.0$ in this work. The results in figure 3(a) of this work and figure 2(b) of reference [5], that $T_{\mathrm{i}\perp} > T_{\mathrm{i}\parallel}$ gives an overall stabilizing effect, are similar. However, the numerical results with different s value in this work show that, (1) increasing Λ_i does not give a small destabilization effect on the long-wavelength modes; (2) there is not a significant stabilizing effect on the short-wavelength modes when $T_{\mathrm{i}\perp} > T_{\mathrm{i}\parallel}$. The opposite is in reference [5], where the magnetic shear effect is neglected. In addition, the

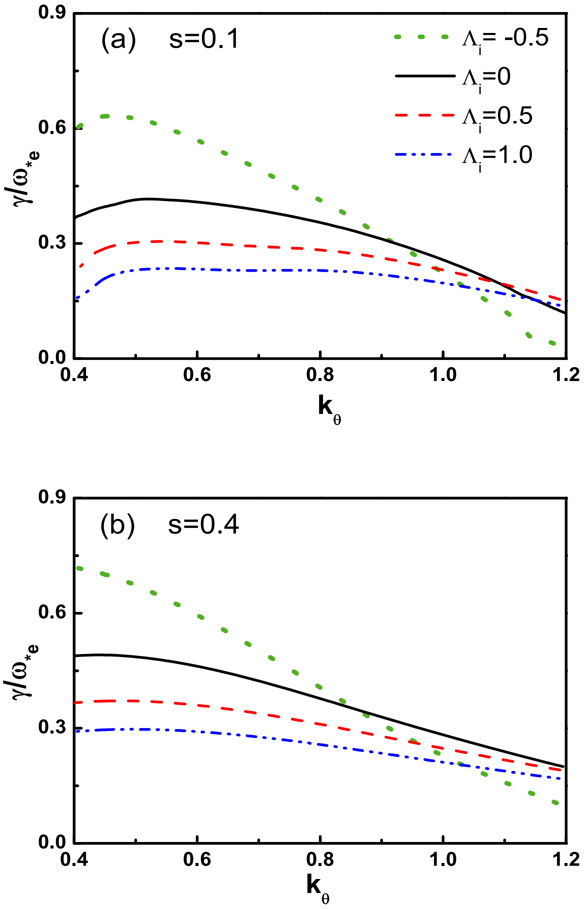

Figure 2. Normalized growth rate γ/ω_{*e} vs k_θ for different s and Λ_i. The other parameters are $\eta_\perp = \eta_\| = 3.0, \varepsilon_n = 0.2, q = 1.5, \tau_e = 1.0$.

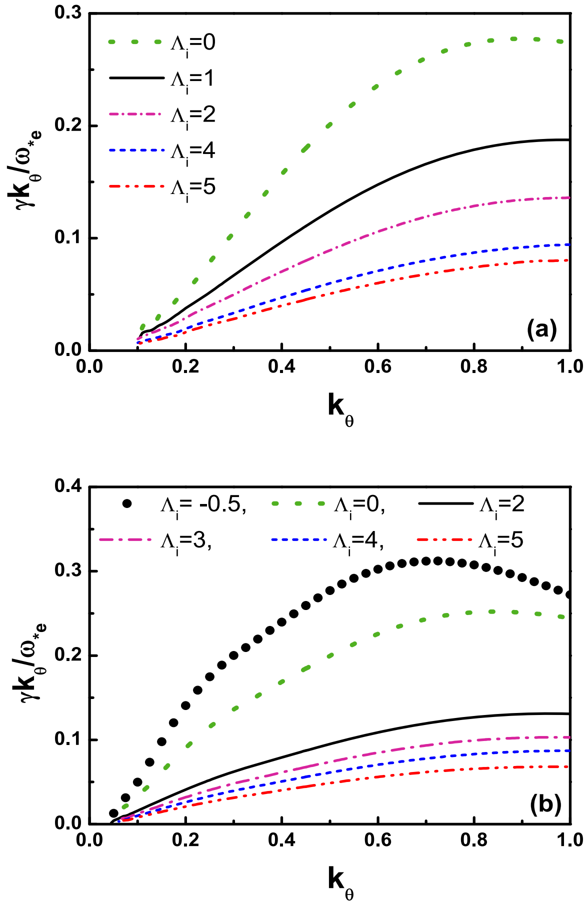

Figure 3. Normalized growth rate $\gamma k_\theta/\omega_{*e}$ vs k_θ for different Λ_i. The other parameters are $\eta_\perp = \eta_\| = 3.0, \varepsilon_n = 0.1, \tau_e = 1.0, q = 1.5$, $s = 0.4$ in (*a*) and $s = 1.0$ in (*b*).

stabilizing effect on the short-wavelength modes, presented under the condition of $\Lambda_i < 0$, is not mentioned in any previous works, because most of studies are focus on the $T_{i\perp}/T_{i\|} > 1$ region.

Now, we consider the normalized growth rate and real frequency versus magnetic shear under different temperature anisotropy conditions. From figure 4, one can see that the growth rate increases, reaches a maximum at about $s \approx 0.65$ and $s \approx 0.5$ for positive and negative Λ_i, respectively, and then decreases for a fixed Λ_i when s increases. The anisotropy in the direction of $T_{i\perp} > T_{i\|}$ also gives an overall stabilizing effect whatever the magnetic shear value is. In addition, a significant stabilizing effect is presented when $T_{i\perp} < T_{i\|}$, the effect of magnetic shear in negative Λ_i region is stronger than that in positive Λ_i region in a toroidal system.

3.2. Synergy of magnetic shear and ion temperature gradient anisotropy

Now, we investigate the effect of the other anisotropy, the ion temperature gradient anisotropy, keeping the anisotropy of ion temperature. Shown in figure 5 is the normalized growth rate as functions of $\eta_\perp$ or $\eta_\|$. Three cases, $\Lambda_i = -0.5, 0$ and 1, corresponding to higher parallel temperature, isotropic temperature and higher perpendicular temperature are given, respectively. The magnetic shear is set to 0.1, corresponding to weak magnetic shear regime which is more interesting. The results confirm the previous work that both $\eta_\|$ and $\eta_\perp$ can drive the ITG instability even in such a weak magnetic shear regime. In addition, the growth rate increases with $\eta_\perp$ for a fixed $\eta_\|$ much faster than it does with $\eta_\|$ for a fixed $\eta_\perp$, which means that the driving force from perpendicular temperature gradient is stronger than that from parallel one. This conclusion can be seen by comparing the growth rates at zero points of the abscissa in figures 5(*a*) and (*b*), where the growth rate of ITG mode at $\eta_\| = 0$ and $\eta_\perp = 1$ and 3 is higher than that at $\eta_\perp = 0$ with $\eta_\| = 1$ and 3 for the three Λ_i values. It presents that the perpendicular temperature gradient makes ITG mode more unstable if ITG mode is driven by a temperature gradient of same value in one direction only. This conclusion is partially broken in the negative Λ_i region, where the slope of the growth rate in part (*b*) for fixing $\eta_\perp$ while changing $\eta_\|$ is slightly larger than that in part (*a*) for fixing $\eta_\|$ while changing $\eta_\perp$. In addition, the two red curves in figure 5(*a*) or figure 5(*b*) still maintain the original trend, even the temperature gradient in either direction reaches a large value. This indicates that the driving force of the parallel temperature gradient is enhanced when $\Lambda_i < 0$. We can get some theoretical insight from equation (3) for this observation. Roughly speaking, the influence of

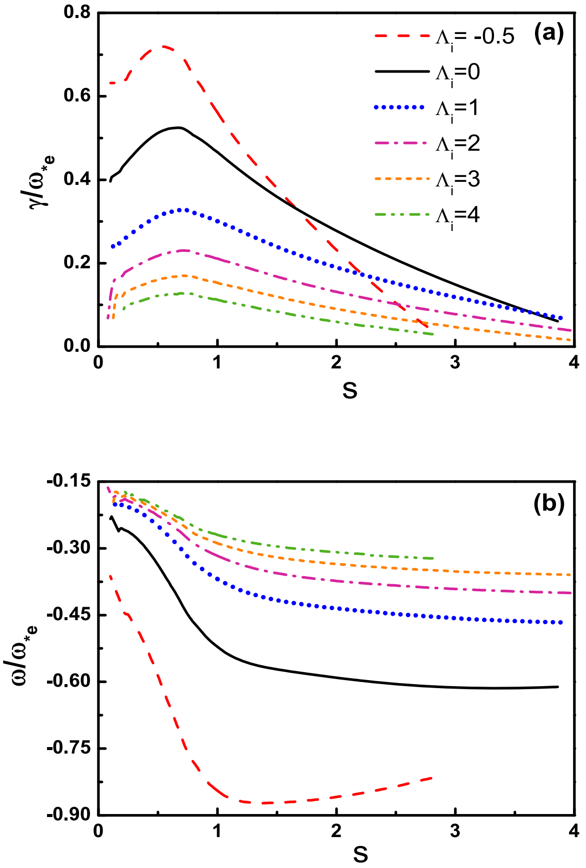

Figure 4. Normalized growth rate and real frequency vs magnetic shear for different Λ_i. The other parameters are $\eta_\perp = \eta_\parallel = 3.0, \varepsilon_n = 0.2, \tau_e = 1.0, q = 1.5$ and $k_\theta = 0.45$.

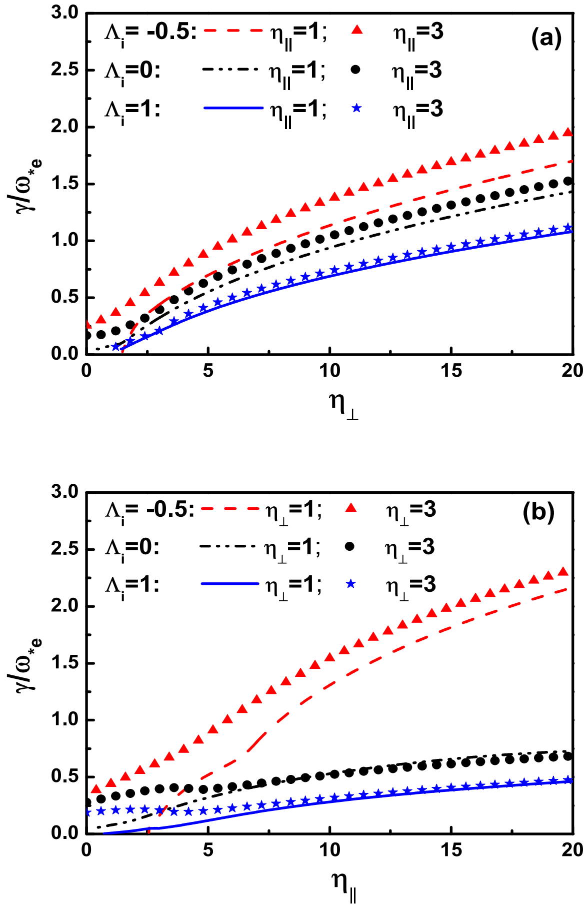

Figure 5. Normalized growth rate versus ion temperature gradient in one direction, $\eta_\perp$ (*a*) and $\eta_\parallel$ (*b*), for different Λ_i and the temperature gradient in another direction. The other parameters are $\varepsilon_n = 0.2, \tau_e = 1.0, q = 1.5, k_\theta = 0.45$ and $s = 0.1$.

parallel and perpendicular temperature gradient mainly embodied in finite Larmor radius and wave-particle resonance, respectively, besides the diamagnetic drift which is common for both. There are three terms in kernel function $K(k, k')$, which depend on the parameter of ion temperature anisotropy Λ_i, i.e. a, λ and ζ. A simple mathematical manipulation indicates that the involvement of Λ_i in λ is virtual. a and ζ are closely related with Λ_i. On the other hand, the parameter $\eta_\parallel$ appears at two places, i.e. $-\eta_\parallel/2$ and $\eta_\parallel(k-k')^2/4a\zeta$. The first term represents the drift while the second term does the wave particle interaction, including Landau damping. Now it is clear that ζ is enhanced in the weak magnetic shear region when Λ_i is negative.

The results for ion temperature gradient anisotropy with magnetic shear are presented in figure 6. The normalized growth rates versus s are given for $\eta_\parallel = 3$ and $\eta_\perp = 3$ in figures 6(*a*) and (*b*) respectively. The results show that neither temperature anisotropy nor temperature gradient anisotropy can change the stabilizing effect of magnetic shear on ITG mode. Moreover, the stabilizing effect of s is stronger for negative Λ_i values. When $T_{i\perp} > T_{i\parallel}$ ($\Lambda_i > 0$), the influence of temperature anisotropy becomes weaker and weaker, showing the blue and red lines or the blue and red symbols getting closer and closer when s is large enough. Comparing the blue dash-dotted lines ($\Lambda_i = 1$) in figures 6(*a*) and (*b*), the line with $\eta_\parallel = 3$ and $\eta_\perp = 2$ in figure 6(*a*) is slightly faster than that with $\eta_\perp = 3$ and $\eta_\parallel = 2$ in figure 6(*b*) to reach zero. This further confirms the above conclusion that the driving force of temperature gradient in the perpendicular direction is stronger than that in the parallel direction when $\Lambda_i > 0$. On the other hand, for the black solid lines ($\Lambda_i = -0.5$), the growth rate in figure 6(*a*) also tends to zero faster than that in figure 6(*b*). This indicates that stabilization effect of the magnetic shear is enhanced in the high shear region when $\Lambda_i = -0.5$.

3.3. Synergy of ion temperature gradient threshold value

Here, the influence of ion temperature and temperature gradient anisotropy on the ion temperature gradient threshold for excitation of ITG mode is discussed, and compared with the results for the isotropic plasmas.

Firstly, the effect of ion temperature anisotropy on the threshold is considered only, it means $\Lambda_i \neq 0$ and $\eta_{i\perp} = \eta_{i\parallel}$. Figure 7 shows the relation between the ion temperature gradient threshold $\varepsilon_{Tc} = \varepsilon_n/\eta_{ic}$ and density gradient parameter ε_n, where the magnetic shear effect is also considered. For isotropic plasma, it is generally believed that the threshold of ion temperature gradient in tokamak plasmas is about 1.0, ($\eta_{ic} \sim 1$), and increases with the flattening of density distribution. In this figure, two critical curves obtained from

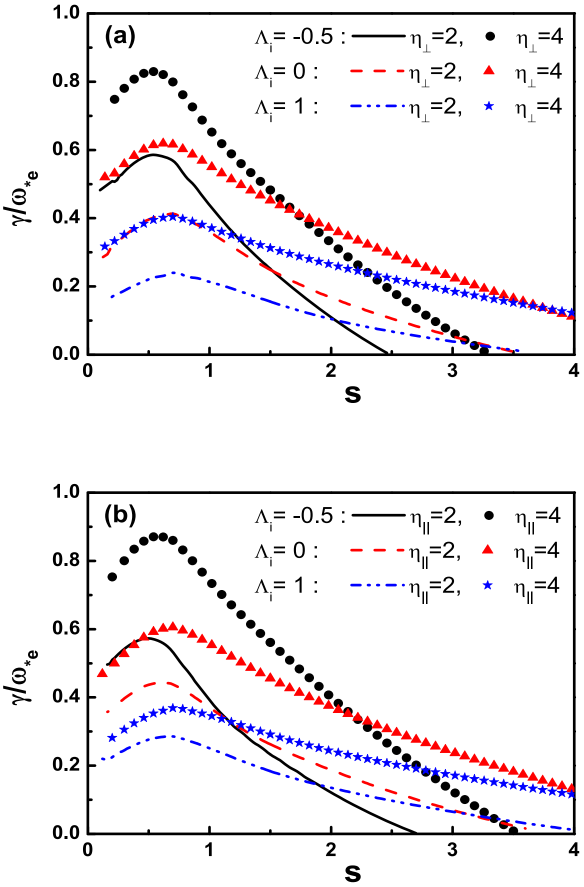

Figure 6. Normalized growth rate versus magnetic shear for different Λ_{i} and temperature gradient in one direction. The other parameters are $\varepsilon_n = 0.2, \tau_{\mathrm{e}} = 1.0, q = 1.5, k_\theta = 0.45$, $\eta_{\parallel} = 3$ in part (*a*) and $\eta_{\perp} = 3$ in part (*b*).

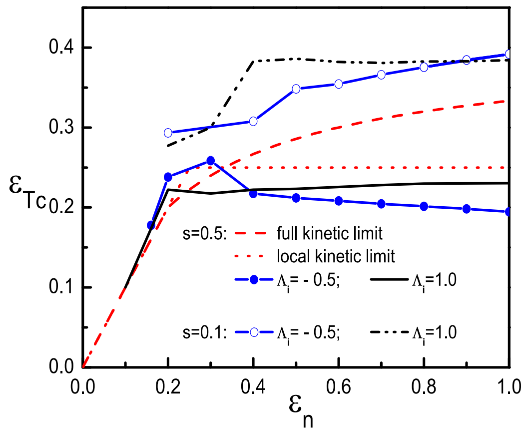

Figure 7. Stable and unstable regions in ε_{Tc} vs ε_n space for different s and Λ_{i} values. The other parameters are $\tau_{\mathrm{e}} = 1.0, q = 1.5, k_\theta = 0.45$ and $\eta_{\parallel} = \eta_{\perp}$.

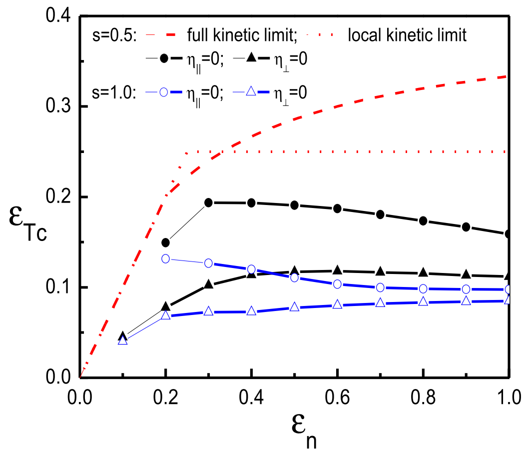

Figure 8. Stable and unstable regions in ε_{Tc} vs ε_n space. The other parameters are $\tau_{\mathrm{e}} = 1.0, q = 1.5, k_\theta = 0.45$ and $\Lambda_{\mathrm{i}} = 0$.

different mode in a tokamak are plotted [11], where the red dotted line presents the threshold values under local kinetic limit $\eta_{\mathrm{ic}} = 1$ and $\eta_{\mathrm{ic}} = (1 + 1/\tau)2\varepsilon_n$ when $\varepsilon_n < 0.25$ and $\varepsilon_n > 0.25$, respectively. The red dash line is the results of full kinetic limit $\eta_{\mathrm{ic}} = 1$ and $\eta_{\mathrm{ic}} = 1 + 2.5(\varepsilon_n - 0.2)$ when $\varepsilon_n < 0.2$ and $\varepsilon_n > 0.2$, respectively. The threshold curve divides to the stable region (above) and the unstable region (below). Regardless of the effect of magnetic shear, for a fixed value $s = 0.5$, (blue line with solid circles, black solid line, red dotted line and red dashed line), the numerical results show clearly that the ion temperature anisotropy makes ITG mode stable in most of ε_n region, corresponding to decrease the ε_{Tc} values. This is consistent with the previous conclusion, that the positive or sufficiently large negative Λ_{i} has a stabilization effect on ITG mode, owing to that ion Landau damping plays an important role. When the density profile is very steep, such as $\varepsilon_n < 0.2$, these four lines are almost the same, meaning that density gradient has significant effect on ITG mode in the smaller ε_n region. On the other hand, whether the perpendicular temperature or the parallel temperature is large, the bigger magnetic shear is helpful to stabilize the ITG mode, and the threshold corresponding to $s = 0.5$ is lower than the threshold of $s = 0.1$, even the previous result has shown that the magnetic shear $s = 0.1$ stabilizes ITG mode more effectively. This shows that the ion temperature anisotropy does not change the qualitatively effect of magnetic shear on the threshold for excitation of ITG mode.

Secondly, the effect of ion temperature gradient anisotropy on the ε_{Tc} is plotted in figure 8, where the ion temperature anisotropy is neglected. Since it is assumed that the temperature gradient in one of two directions, perpendicular or parallel, is zero ($\eta_{\perp} = 0$ or $\eta_{\parallel} = 0$), it means that a driving effect on ITG mode is reduced, a larger value in the other direction ($\eta_{\perp}$ or $\eta_{\parallel}$) is required, the corresponding temperature threshold is lower than the isotropic result. The same results can be obtained that the large magnetic shear can effectively expand the stable region of the ITG mode for most of the ε_n region. In the small ε_n region ($\varepsilon_n < 0.2$ and $\varepsilon_n < 0.1$ when $\eta_{\parallel} = 0$ and $\eta_{\perp} = 0$, respectively), the very steep density gradient increases the drift instability, the thresholds corresponding to different magnetic shear are close to each other. In addition, considering the temperature gradient anisotropy, the driving force coming from the perpendicular temperature gradient is easier to excite ITG mode. As is shown that the threshold $\eta_{\perp Tc}$ at $\eta_{\parallel} = 0$ (black line with circles) is higher than $\eta_{\parallel Tc}$ at $\eta_{\perp} = 0$ (black line with triangles). This is consistent with the above results that the perpendicular temperature gradient has a stronger destabilizing effect.

4. Conclusion and discussion

The gyrokinetic integral eigenmode equation for studying micro-drift instabilities is applied to analyze the ITG modes in toroidal plasmas, where the ion temperature and its gradient are anisotropic. The full kinetic mechanism of the ions, such as finite Larmor radius, magnetic gradient and curvature drifts, are taken into account.

The main conclusion includes: (1) ion temperature anisotropy reduces the ITG instability when the perpendicular temperature is higher than the parallel temperature, corresponding to $\Lambda_\mathrm{i} > 0$, since ion Landau damping plays an important role to stabilize ITG mode. However, ITG mode cannot be completely stabilized even for a large positive Λ_i. On the other hand, when the parallel temperature is high enough, corresponding to a large negative Λ_i, ITG mode can also be stabilized since high parallel temperature corresponds to high parallel velocity, which also enhances the Landau damping effect. (2) The driving force of ion temperature gradient in the perpendicular direction is stronger when $\Lambda_\mathrm{i} > 0$, while that of the parallel temperature gradient is relatively more visible when $\Lambda_\mathrm{i} < 0$. The analysis of the Eigen-equation shows that Λ_i mainly acts on the parallel temperature gradient, corresponding to reducing and enhancing $\eta_\parallel$ effect for $\Lambda_\mathrm{i} > 0$ and $\Lambda_\mathrm{i} < 0$, respectively. (3) The synergy effect of magnetic shear and anisotropy of ion temperature and its gradient presents the overall stabilizing effect of magnetic shear on ITG mode in a toroidal plasma, which is different from the destabilizing effect of $s \geqslant 0.4$ in a slab plasma. Compared with the case of non-magnetic shear, the effects of big Λ_i and s do not give a destabilization/stabilizing effect on the long-wavelength/short-wavelength modes when $\Lambda_\mathrm{i} > 0$, respectively. On the other hand, the inhibition of high magnetic shear becomes stronger when $\Lambda_\mathrm{i} < 0$. (4) By considering the temperature gradient threshold for excitation of ITG mode, the positive or sufficiently large negative Λ_i also reduces the unstable area of ITG mode, and the unstable region of perpendicular temperature gradient threshold is greater than that of the parallel temperature gradient threshold. This is still caused by ion Landau damping. But $s = 0.1$ (the weak magnetic shear) is no longer a special value for exciting ITG mode. It still obeys the law that the greater the magnetic shear, the larger the stable region is.

In this paper, the circular section and large-aspect-ratio geometry are assumed. However, many real tokamaks are of non-circular cross-section, the shaping of magnetic flux surfaces is important on turbulence in tokamak edge plasmas. Some simulation code HELENA [19], GEM [20], etc can study the elongation, triangularity and a divertor configuration. Their work shows that transport is mainly reduced by local magnetic shear and an enhancement of sheared f zonal lows induced by elongation and X-point shaping [21]. The non-circular cross-section will be our next step of work to study the ITG and micro-drift instabilities in anisotropic plasma. It will be more useful for real tokamak experiments.

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References

[1] Horton W. 1999 *Rev. Mod. Phys.* **71** 735
[2] Tang W.M. 1978 *Nucl. Fusion* **18** 1089
[3] Migliuolo S. 1988 *Phys. Lett.* A **131** 373
[4] Mathey O. and Sen A.K. 1989 *Phys. Rev. Lett.* **62** 268
[5] Kim J.Y., Horton W., Choi D.I., Migliuolo S. and Coppi B. 1992 *Phys. Fluids*B **4** 152
[6] Song H. and Sen A.K. 1993 *Phys. Fluids* B **5** 2806
[7] Dong J.Q., Long Y.X. and Avinash K. 2001 *Phys. Plasmas* **8** 4120
[8] Dong J.Q., Horton W. and Kim J.Y. 1992 *Phys. Fluids* B **4** 1867
[9] Lee Y.C., Van Dam J. and Glasser A. 1977 *Proc. of the Finite Beta Theory Workshop* (Washington, DC: U.S. Department of Energy) https://www.osti.gov/biblio/5984608 p 55, 93
[10] Courant R. and Hilbert D. 1953 *Methods of Mathematical Physics* (New York: Wiley)
[11] Romanelli F. 1989 *Phys. Fluids*B **1** 1018
[12] Delves L.M. and Walsh J. 1974 *Numerical Solution of Integral Equations* (Oxford: Clarendon)
[13] Gao Z. *et al* 2003 *Phys. Plasmas* **10** 774
[14] Liu Songfen *et al* 2009 *Phys. Plasmas* **16** 774
[15] Dong J. 2018 *Plasma Sci. Technol.* **20** 094005
[16] Liu S., Guo S.C. and Dong J.Q. 2010 *Phys. Plasmas* **17** 052505
[17] Liu S.F., Guo S.C., Zhang C.L., Dong J.Q., Carraro L. and Wang Z.R. 2011 *Nucl. Fusion* **51** 083021
[18] Holod I. and Lin Z. 2013 *Phys. Plasmas* **20** 032309
[19] Cesario R *et al* 2013 *Plasma Phys. Control. Fusion* **55** 045005
[20] Scott B.D. 2005 *Phys. Plasmas* **12** 102307
[21] Alexander Kendl and Scott Bruce D. 2006 *Phys. Plasmas* **13** 012504